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Money is Memory*

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ABSTRACT

This paper examines the sets of feasible allocations in a large class of economic environments in which commitment is impossible (the standard definition of feasibility is adapted to take account of the lack of commitment). The environments feature either *memory* or *money*. Memory is defined as knowledge on the part of an agent of the full histories of all agents with whom he has had direct or indirect contact in the past. Money is defined as an object that does not enter preferences or production and is available in fixed supply. The main proposition proves that any allocation that is feasible in an environment with money is also feasible in the same environment with memory. Depending on the environment, the converse may or may not be true. Hence, from a technological point of view, money is equivalent to a primitive form of memory.

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I. Introduction

At its heart, economic thinking about fiat money is paradoxical. On the one hand, such money is viewed as being an intrinsically useless object that does not enter preferences or production functions. But at the same time, these barren tokens are a *technological* innovation: their presence allows societies to implement allocations that would otherwise not be achievable. The purpose of this paper is to uncover exactly what aspect of technology is being enhanced by the presence of money.

To this end, I examine a wide class of economic environments. The environments are characterized by a matching process that partitions agents into physically and informationally separated groups at each point in time. Within each group, there are no gains to trade. I show that the class of environments includes the setups underlying the overlapping generations, turnpike, and random matching models. The environments share an additional friction beyond the physical restrictions on resource reallocation imposed by the matching process: agents are unable to *commit* themselves to a particular allocation of resources. Using the mechanism design approach[1] of Myerson (1979), the paper provides a rigorous definition of what allocations are feasible given the presence of this friction; I term these allocations *incentive-feasible*.

Within the class of environments under consideration, I show that adding money may expand the set of incentive-feasible allocations. However, I also show that the set of incentive-feasible allocations can be expanded by adding collective *memory* to these environments. Here, memory is defined to be a historical record which reports to any agent at any date the full

[1]Huggett and Krasa (1996) also use a mechanism design approach to analyze money. However, I emphasize alternative incentive issues - namely those involved with lack of commitment - and consequently obtain much more general results.

histories of all agents with whom he has had direct or indirect contact in the past.[2] The main result of the paper is that in all of these environments, the set of incentive-feasible allocations generated by adding *memory* contains the set of incentive-feasible allocations generated by adding *money*. In this sense, in all of these environments, money is merely a primitive form of memory.

There is a simple reasoning behind the main proposition. John and Mary meet. John has apples and wants bananas. Mary wants apples but doesn't have bananas. In monetary economies, this problem is solved by Mary's giving John money in exchange for apples. John then uses the money to buy bananas from Paul; if John doesn't give the apples to Mary, John doesn't get the money and can't buy the bananas from Paul.

But of course the money itself is intrinsically useless. In terms of the reallocation of intrinsically valuable resources, we can think about the situation as being one in which John is considering making Mary a *gift* of apples. If he makes the gift, Paul will give him bananas in the future; if he doesn't make the gift, Paul won't give him the bananas. The money that John receives from Mary is merely a way of letting Paul know that John has fulfilled his societal obligations and given Mary her apples.

Thus, if we account for the fact that money itself is useless, monetary allocations are merely large interlocking networks of gifts. The point of this paper is to show that these same reallocations of resources are feasible if agents knew the past history of all actions: Paul could react to different histories of gifts on John's part in the same way that he reacts to John's having

[2]Note that this version of memory does not allow an agent in a given period to access the past histories of his *future* trading partners.

different amounts of money. It follows that any function performed by money can be provided by an ability to access the pasts of one's trading partners, their trading partners, and so on.[3]

Others have noted that, as emphasized in this paper, fiat money helps to keep track of past actions. In early work along these lines, Ostroy (1973) discusses the recordkeeping role of money in a model in which static competitive allocations are implemented using decentralized pairwise exchange. Lucas (1980) uses a back-of-the-envelope calculation to argue that money is a much cheaper means of keeping track of past reallocations of resources than other means of recordkeeping. In a random matching model, Aiyagari and Wallace (1991) show how money becomes redundant when agents have access to a historical record of all actions taken in past matches.

The contribution of this paper over this past work is to emphasize both the *singularity* and the *generality* of money's recordkeeping role. I show that the technological benefits of money are completely subsumed by the technological benefits of just one type of technological innovation: memory. Moreover, I show that this result is true in an extremely broad class of

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[3]Townsend (1987, 1989, 1990) considers two period environments in which endowments are private information. In the Pareto optimal allocations of these environments, agents who reveal that they have high endowment realizations in the first period receive more consumption in the second period than those who announce low endowment realizations in the first period. This is easy to accomplish if the planner can keep track of agents' identities over time. If not, though, then the planner in the first period can give the agent with good shocks some kind of nonreplicable token that they can reveal in the second period to show they are supposed to receive extra consumption. Thus, in the second period, there is an implicit exchange of barren tokens for consumption - and this exchange is essential in achieving optimal allocations.

This work is helpful in understanding certain types of financial arrangements. However, as Townsend emphasizes, it cannot be interpreted as an explanation of fiat money. In the second period of Townsend's models, agents give up consumption to those with tokens because they face some dire punishment if they don't. In contrast, the essential ingredient of fiat money is that individuals are willing to *voluntarily* give up real resources for intrinsically useless tokens. The tokens in Townsend's model are a type of credit and not a type of fiat money.

environments.

In the next section, I describe the class of environments under study, and show that it nests the standard overlapping generations, turnpike, and random matching models. In Section III, I define memory and money, and what an incentive-feasible allocation is. In Section IV, I prove the major result that memory technologically dominates fiat money. In Section V, I discuss the technological roles of bonds and credit cards, and compare them to the technological role of money. Finally, in Section VI, I conclude.

II. A Class of Physical Environments

1. General Discussion

In this section, I describe a class of physical environments that includes the settings underlying three main paradigms of fiat money (overlapping generations, turnpike, and random matching). In this set of environments, agents are divided each period into physically separated groups. The division is extreme in the sense that if two agents are in the same group, then the first agent has not had direct or indirect contact with any of the individuals with whom the second agent has had indirect or direct contact.

To be more specific, consider a world with a finite number K of perishable goods. There is a set of agents indexed by ω, where ω lies in $\Omega \subseteq [0,1]$. Different agents can live for different lengths of time. Let Ω_t denote the indices of agents who are alive at the beginning of period t. I assume that Ω_t either has positive Lebesgue measure, is a countably infinite subset of [0,1], or a finite subset of [0,1].

In period t, a generic agent ω has a momentary utility function $u^\omega\colon \Re^K \rightarrow [\underline{u},\overline{u}]$. All agents have the same discount factor β. Hence, the generic agent ω, in period t, has preferences over streams of consumption that are representable by the utility function

$$u^{\omega}(c_t) + E_t \, \Sigma_{r=1}^{T(\omega)} \beta^r u^{\omega}(c_{t+r})$$

where c_{t+r} is some element of $\Re^K$. Here, $T(\omega)$ is the number of years left in agent ω's life and may be infinity; E_t represents an expectation conditional on information available to the agent in period t.

At each date, a generic agent ω has a nonnegative endowment $y_t^{\omega} \in \Re^K$ and a technology $Y_t^{\omega} \subseteq \Re^K$ with which to produce more goods. I assume that $0 \in Y_t^{\omega}$, and that agent ω can freely dispose of any goods; hence, if $y \in Y_t^{\omega}$, then $y' \in Y_t^{\omega}$ for any $y' \leq y$. The characteristics of each agent ω (that is, endowments, preferences, technologies, and birth and death dates) are common knowledge among all agents.

The state of the world is defined by a stochastic process s_t which determines a partition of the agents who are alive. I term an element of this partition a *match*; for technical reasons, it is convenient to assume that each match contains a finite number of agents. The partition determined by s_t represents both the physical and informational separation of agents: thus, no transfers of any goods can take place among agents who are in different matches, and agents do not know what has taken place in another match. Similarly, the state s_t is *not* common knowledge. Rather, at time t, a given agent observes only the indices of all agents in his current and past matches. I assume that s_t is independent of the past in the sense that $\Pr(s_t | s_{t-1}, s_{t-2}, \ldots, s_1)$ is the same for all past histories $(s_{t-1}, \ldots, s_1)$ (but note that s_t may be deterministic and/or nonstationary).

Given the physical separation across matches, the following is a natural definition of feasible allocations.

Definition 1: For every match of J agents at date t, an *allocation* specifies a K-dimensional vector c_t^j for each agent j; an allocation is *feasible* if, in every match, every component of c_t^j is nonnegative and $(\Sigma_{j=1}^{J} c_t^j - \Sigma_{j=1}^{J} y_t^j) \in \Sigma_{j=1}^{J} Y_t^j$.

The definition of feasibility respects the restriction that there is no way to transfer goods from one match to another.

I assume that the matching process has the following feature that ensures a severe informational separation among the agents. Let $P_t(\omega)$ be the agents who are in agent ω's match with ω in period t (including ω himself), and then define $Q_t(\omega)$ recursively as

$$Q_1(\omega) = P_1(\omega)$$

$$Q_t(\omega) = \bigcup_{\nu \in P_t(\omega)} Q_{t-1}(\nu).$$

(If ω is not alive in period t, then $P_t(\omega)$ is empty.) Thus, $Q_t(\omega)$ consists of agent ω and his trading partners in period t, all of their trading partners in period (t-1), all of those people's trading partners in period (t-2), and so on. I assume that the matching process is such that

(R1) With probability one, if ν and ω are matched at time t, $Q_{t-1}(\omega) \cap Q_{t-1}(\nu) = \varnothing$.

The assumption (R1) means that agent ω's past matches reveal nothing about agent ν's past matches because there is no possibility of any direct or indirect contact between the two agents before the current match.

I impose another restriction on the matching process. First, for any agent ω, define his autarkic utility within period t to be

$$u^{\omega}_{aut,t} \equiv \max_{c} \ u^{\omega}(c)$$

$$\text{s.t. } (c - y^{\omega}_{t}) \in Y^{\omega}_{t}.$$

I assume that the matching process satisfies

(R2) (No static gains to trade) In any match, there exists no feasible allocation $(c^j_t)^J_{j=1}$ of resources within the match that Pareto dominates autarky in the sense that $u^j(c^j_t) \geq u^j_{aut,t}$ for all j, with $u^j(c^j_t) > u^j_{aut,t}$ for some j.

As we shall see, this restriction serves to highlight the role of money and memory by essentially eliminating all possibility for trade when they are absent.

From now on, I use the term "environment" to refer to an environment as discussed above. The following are familiar examples of environments in this specific sense.

2. Overlapping Generations

Consider an overlapping generations economy in which every agent lives two periods. There are J agents in each cohort. Agents are each endowed with y_1 units of a perfectly divisible consumption good when young and y_2 units of consumption when they are old; the consumption good is not storable. (Here, technologies are equal to $\{y | y \leq 0\}$). The young agents have preferences over current consumption (c_y) and future consumption (c_o) that are representable by the utility function

$$u(c_y) + u(c_o)$$

where u is strictly increasing. The old agents prefer more consumption to less.

Label the agents in each cohort by numbers from 1 to J. Then think of the matching process as separating the 2J agents alive at each date by partitioning them into pairs consisting of the old agent j and the young agent j. The set Q_{t-1}(young agent j) is empty, so (R1) is trivially satisfied. Also, it is clear that because there is only one good, there are no static gains to trade.

3. *Turnpike*

As in Townsend (1980), consider a world with an infinite number of trading posts located at the integer points along the real line. In period one and in every period thereafter, at each trading post there are J "stayers" and J "movers". At the end of period t, the movers move 2^{t-1} trading posts to the right; the stayers stay at their current trading posts. (The unorthodox size of the shifts is in order to make the matching process consistent with (R1).) The agents in each cohort at each trading post are indexed using the natural numbers from 1 to J.

In period t, stayers are endowed with one unit of consumption if t is odd and zero units of consumption if t is even; the movers are endowed with zero units of consumption if t is odd and one unit of consumption if t is even. (As above, technologies are equal to $\{y \mid y \leq 0\}$.) Consumption is perishable. In period t, each type of agent has preferences over current and future consumption representable by the function

$$\Sigma_{s=0}^{\infty} \beta^s u(c_{t+s}),\ 0 < \beta < 1$$

where u is strictly increasing, strictly concave, and bounded from above and below.

Suppose the matching process pairs stayer j with mover j at each trading post. Consider the mover j who started life at trading post 0; in period t, he arrives at trading post $2^{t-1} - 1$. It is tedious but simple to show that for this agent, Q_t contains all stayers and movers labelled j who began life at posts $\{0, 1, \ldots, 2^{t-1} - 1\}$. At the same time, the set Q_t for mover j at trading post $(2^t - 1)$ contains all stayers and movers labelled j who began life at posts $\{2^{t-1}, \ldots, 2^t - 1\}$. Hence, the matching process satisfies (R1). Because there is only one good at each post, the process also satisfies (R2).

4. Random Matching

The following is a simplified version of the environment underlying Trejos and Wright (1995). Consider an environment in which $\Omega = [0,1]$; there are three types of agents and there is a measure 1/3 of each type. There are also three types of nondurable, indivisible, nonstorable goods. In each period, a type i agent can produce some nonnegative amount of good i less than or equal to $\bar{y}$. Type i agents have momentary utility $u(c_{i+1}) - y_i$, where c_{i+1} is consumption of good $(i+1)$ and y_i is production of good i. (The utility function u is assumed to be strictly increasing and strictly concave.) The agents live forever and discount utility using the discount factor β.

In the more general notation developed above, we can think of there being four goods: the three perishable goods and time. The endowment vector for each agent has the form $y_t^z = (0,0,0,\bar{y})$. A type 1 agent has technology

$$Y_t^1 = \{(y_1, y_2, y_3, y_4) \in \Re^4 | y_1 \leq -y_4, y_2 \leq 0, y_3 \leq 0\}.$$

Note that the technology satisfies free disposal. The upper bound on production in any allocation is a consequence of a nonnegativity constraint on the agent's allocation of time.

In this environment, the matching process randomly divides the agents into pairs; a given individual is equally likely to be matched with any of the three types of agents. Note that because $Q_t(\omega)$ is a set of measure zero for all t and ω, condition (R1) is trivially satisfied. Also, the structure of agent preferences and technologies guarantees that condition (R2) is satisfied as well.

Thus, the class of physical environments studied in this paper embeds the standard paradigms of modern monetary theory.

III. Incentive-Feasible Allocations

In this section, I augment the above physical environments first with money and then, instead, with memory. Having done so, I define *no-commitment* trading mechanisms, and prove a "revelation principle" that shows that any equilibrium outcome of any no-commitment mechanism is an equilibrium outcome of a particular *direct* mechanism. I use this result to describe a notion of *incentive-feasibility* for three types of environments: ones without money or memory, ones with money, and ones with memory.

1. Money and Memory

First, suppose there is another good that is durable that does not enter preferences or production; call this good money. The per capita supply of money is fixed at M over time. At the end of any period, because of the physical properties of money, any agent's holdings of this durable good are restricted to lie in the finite[4] set $X \subset [0, \infty)$; I assume that X contains $\{0\}$. Any agent born after date 1 begins life with zero units of money.

The initial stock of money can be distributed across agents in Ω_1 (that is, agents alive at the beginning of period 1) in two different ways. The first method specifies a function $m_0:\Omega_1 \to X$ such that the per capita[5] level of $m_0(\omega)$ is M. Here, the initial level of moneyholdings for each agent is a deterministic function of his identity.

The second method only works if Ω_1 is countably infinite or a set of positive Lebesgue measure. Define μ to be any measure over X such that $E(\mu) = M$; for each person in Ω_1, their initial moneyholdings are drawn independently from μ. The measure μ is common knowledge among all agents (not just those alive at the beginning of period 1). Note that both of these methods of distributing money across agents in period 1 imply that an agent has independent (possibly degenerate) priors over his compatriots' initial moneyholdings.

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[4]The finiteness of X is technically convenient, but I do not believe that it is essential in what follows. (In any event, money is generally discrete and it is natural to think there are upper bounds to how much of it can be stored from one period to the next.)

[5]The term "per-capita" will have different meanings depending on the size of Ω_t. In particular, let $x_t(\omega)$ be some real-valued characteristic associated with each agent ω in Ω_t. Then, the per capita level of this characteristic, X_t, is given as follows:

If Ω_t is finite, $X_t = \{\Sigma_{\omega\epsilon\Omega_t} x_t(\omega)\}/|\Omega_t|$.
If $\Omega_t = \{\omega_1, \omega_2, \omega_3, \ldots\}$, $X_t = \lim_{J\to\infty} J^{-1}\Sigma_{j=1}^{J} x_t(\omega_j)$.
If Ω_t is a subset of [0,1] with positive Lebesgue measure, then
$$X_t = \int_{\Omega_t} x_t(\omega)\, d\omega.$$

Using per-capita in these different ways is standard. (Note that the last definition implicitly assumes that x_t is Lebesgue integrable as a function of ω.)

Just as with the K perishable goods, allocations of money must obey the physical trading restrictions imposed by the matching process.

Definition 2: For every match of J agents at date t, a *money allocation* specifies a scalar m_t^j for each agent j; a money allocation is *feasible* if, in every match, $m_t^j \in X$ and $\Sigma_{j=1}^J m_{t-1}^j \geq \Sigma_{j=1}^J m_t^j$.

Note that allocations of money also must obey the individual feasibility restriction that individual moneyholdings lie in X.

Now suppose that instead of adding money, we add memory to the class of physical environments as follows. There is a historical record (spreadsheet) that, for each agent ω, reports ω's past trading partners at each previous date, and the actions of ω and his partners in those matches. Access to the record works as follows: at any date, agent ω can costlessly and instantaneously observe the entries in the record for any individual in $Q_t(\omega)$. However, it is important to note that access is limited in the sense that agent ω is unable to observe the entries in the record for any of his potential future trading partners. The reason for this limitation will become clear later.

2. *No-Commitment Trading Mechanisms*

In this subsection, I first give a precise but general description of *trading mechanisms* (essentially, methods of interactions among the agents that lead to feasible allocations of resources in each match). I then define *no-commitment* trading mechanisms that can be used by societies that do not have any technology of enforcement.

Trading mechanisms have two components. The first component specifies a sequential choice of actions by the various agents in a match. More precisely, suppose agent ω's match in period t contains J agents who begin the period with a vector $\mathbf{m}_{t-1}$ of moneyholdings. Within the period, there are Π subperiods or stages. Each agent ω has an *action set* $A_\pi(t,\omega,s_t, \mathbf{m}_{t-1})$ for each stage π; in each stage, agents choose actions simultaneously and separately. The actions chosen in stage π are common knowledge among the agents in future stages.

The second component of a mechanism is an *outcome function* $f(\mathbf{a};t,\omega,s_t, \mathbf{m}_{t-1})$ that, for each vector $\mathbf{a}$ of agent actions within the previous Π stages, specifies an element of the set of feasible allocations. Agents then receive consumption and produce according to this allocation.

Note that this notion of trading mechanism allows for a wide range of modes of interactions and, in particular, includes the exchange procedures used in the standard monetary models. In the Trejos-Wright random matching model, virtually any kind of finite stage bargaining protocol (including take-it-or-leave-it offers on the part of the consumer) is a trading mechanism. In the overlapping generations and turnpike models, there is a trading mechanism that mimics competitive exchange within each match (even though there are only a finite number of agents). This mechanism has only one stage; the agents' action sets equal their budget sets as calculated using a competitive equilibrium price vector. The outcome function then gives them whatever element of those budget sets that they choose. (Note that the competitive equilibrium price vector, and therefore any budget set, is completely determined by t and ω because these are the sole determinants of endowments, technology and preferences of agents within the match.)

Having defined a trading mechanism in this general fashion, we need to have a notion of equilibrium to know what outcomes can occur when agents follow the rules of play described by the mechanism. Following Abreu, Pearce and Stacchetti (1990), I restrict attention to perfect public equilibria (see also Fudenberg and Tirole (1991, p. 187-188)) in which agents use strategies that do not depend on their private information. I make this restriction for two reasons. First, in most analyses of monetary random matching models, researchers have focused on a subset of perfect public equilibria. Second, because of the restriction (R1) on the matching process and because of the independence of s_t from past information, an agent ω's best response correspondence in a particular match is always independent of his private information. (This second fact is of course not true if agents have private information about their preferences or technologies.)

To use this equilibrium concept, it is important to distinguish between the public information and private information for each agent. This distinction varies across the three types of environments (no memory or money, with money, and with memory). For example, in an environment without money or memory, the public information in a match consists only of the indices of the agents in the match and any actions taken at previous stages within the match. An agent's private information consists of the indices of all agents in his past matches, and the past actions of all agents in his past matches.

In an environment with money, the public information consists of the moneyholdings of the agents, their indices, and any actions taken at previous stages within the match. An individual's private information consists of past reports he has sent and received, the indices and past actions of all agents in his past matches and all moneyholdings of all agents in his past matches. Finally, in an environment with memory, the public information available to agent ω

consists of all actions taken by all agents in $Q_t(\omega)$ in this match and all past matches.

Given these differing notions of information across the different environments, an agent's *strategy* in any environment is a mapping from all of his possible information sets into actions. Then, a (pure strategy) *perfect public equilibrium* (PPE) is a collection of individual strategies such that

(i.) At every information set, an individual's strategy specifies an action that is weakly optimal given that all agents follow their strategies at their current and future information sets.

(ii.) If an individual has two information sets that only differ in his private information, his strategy specifies the same action at those two information sets.

A PPE allocation is one that occurs when all individuals always play the strategies in a PPE.

It is easy to see that in any environment, the first-best allocation specifies a split of resources that depends only on the technologies, momentary utility functions, and indices of the agents within each match. Hence, it is simple to construct a mechanism that uniquely implements this allocation, even in environments without money or memory. The mechanism has a single stage, agents' action sets are singletons, and the outcome function maps this unique vector of actions into the desired allocation.

This simple result tells us that societies need to keep track of the past only if they face some additional friction that interferes with the allocation of resources. The additional friction that I consider is *lack of commitment*: any agent is allowed to refuse at any point in time to go along with a proposed allocation and instead simply produce a consumption vector for himself. This means that the society can only use *no-commitment* trading mechanisms which have the following property:

In any match, for every agent ω in the match, and for all (c^ω, m^ω) such that $c^\omega \in \{y_t^\omega\} + Y_t^\omega$ and $0 \leq m^\omega \leq m_{t-1}^\omega$, there exists an action sequence a_ω such that the allocation $f(a_\omega, \mathbf{a}_{-\omega}; t, s_t, \mathbf{m}_{t-1})$ gives (c^ω, m^ω) to agent ω for all $\mathbf{a}_{-\omega}$.

Here, $\mathbf{a}_{-\omega}$ refers to a vector of action sequences by all other agents except ω. Thus, in a no-commitment trading mechanism, an agent ω always has the ability to choose a sequence of actions that guarantee him a given autarkic consumption vector. Note that autarky is always a PPE of a no-commitment mechanism. Also, the class of no-commitment trading mechanisms includes the competitive exchange mechanism and the bargaining protocols mentioned above.

3. Incentive-Feasible Allocations

The class of no-commitment trading mechanisms is enormous. Fortunately, in this subsection, I show that we can essentially focus on just one no-commitment trading mechanism: the *direct* no-commitment mechanism. The direct mechanism has only one stage. In that stage, each agent ω chooses an element of the set of feasible allocations and an element of $\{y_t^\omega\} + Y_t^\omega$. The outcome function says that if the agents all choose the same feasible allocation, then that allocation is implemented. If they don't all choose the same feasible allocation, then each agent receives the autarkic consumption vector that he chose. Note that this is a no-commitment mechanism because an agent can always choose an action in such a way so as to guarantee a particular autarkic consumption vector for himself.

Given this definition of the direct mechanism, the following proposition is then a version of the revelation principle (Myerson (1979, 1986)).

Proposition 1: A PPE allocation of any no-commitment trading mechanism is a PPE allocation of the direct no-commitment mechanism.

Proof: In Appendix. Δ

In an environment without a technology of enforcement, it is only possible to implement allocations that are equilibria of no-commitment trading mechanisms. Proposition 1 then justifies the following definition of *incentive-feasible* allocations (see Myerson (1979) for a similar use of language).

Definition 3: An *incentive-feasible allocation* of the K perishable goods is one that occurs when all agents always play the strategies of a perfect public equilibrium in the direct no-commitment mechanism.

Let me sum up. In a no-commitment trading mechanism, an agent is always free to choose a consumption vector that he is able to produce on his own. The perfect public equilibrium concept provides a rigorous notion of what outcomes are possible given that agents play according to the rules of a given trading mechanism. Proposition 1 then shows that the set of PPE allocations for any mechanism is a subset of PPE allocations for the direct mechanism; hence, I term incentive-feasible the set of PPE allocations when agents use the direct mechanism, and I think of incentive-feasible allocations as being an exhaustive description of the allocations that the members of a society can achieve in the absence of commitment.

IV. Money is No Better Than Memory

1. General Results

It is instructive to formally demonstrate that without commitment, either money or memory is necessary to allow society to achieve allocations that are better than autarky.

Proposition 2: Suppose the matching process satisfies (R2). Then, in an environment with no memory and no money, there is no incentive-feasible allocation in which all agents receive strictly more ex-ante utility than from simply producing on their own in every period of their lives.

Proof: Suppose the proposition is false, and there is a PPE outcome of the direct mechanism that provides strictly more ex-ante utility to some agent than autarky. In some match, some agent j must have an allocation a_t^j that provides more momentary utility than any element of $\{y_t^j\} + Y_t^j$. Because of the assumption of no static gains to trade, there is another agent k in the match whose allocation a_t^k must provide him with strictly less utility than some element c_t^k of $\{y_t^k\} + Y_t$. If agent k defects to c_t^k, he is made better off today. But agent k's defection is not reflected in any public information about himself in future matches. Hence, it cannot trigger a change in the play in his future matches. Δ

In an environment without money or memory, agents cannot keep track of the past: without commitment, only allocations that offer static gains to trade are incentive-feasible. If the matching process satisfies (R2), no such allocations exist.

It is certainly possible to write down examples of environments such that neither money nor memory expand the set of incentive-feasible allocations. For example, in a finite horizon environment, only autarky is incentive-feasible, regardless of whether money or memory is present. However, there are (well-known) examples of environments in which either money or memory does expand the set of allocations.

Example: Consider the overlapping generations environment discussed in Section II.2. Suppose $y_1 > y_2$. Then, $y^* = (y_1 + y_2)/2$ satisfies

$$u(y^*) + u(y^*) > u(y_1) + u(y_2)$$

$$y^* > y_2.$$

Then, an allocation in which every agent consumes y^* units of output in every period is incentive-feasible if $X = \{0,1\}$ and the initial old agents are each given one unit of money. (It is the equilibrium path of a PPE in which, in every match, the two agents write down that the young agent should give the old $(y^* - y_2)$ units of consumption and the old agent should give up one unit of money; the two agents write down autarky if the old agent ever has less than one unit of money.)

In this environment with memory, the young agent j gives the old agent j ($y^* - y_2$) units of consumption as long as every past young agent j has done so; otherwise, he gives old agent j zero units of consumption. This collection of strategies is a PPE and gives rise to the above allocation.

Note that y^* in the above example is equal to the competitive equilibrium steady-state (when agents are assumed to trade (divisible) money for goods competitively in each period). Because of Proposition 1, the other, usual, notions of equilibrium allocations in the standard paradigms are also incentive-feasible: competitive equilibria in the overlapping generations and turnpike models, and stationary Nash equilibria of various bargaining protocols in random matching models.

The rest of this subsection presents the main theorem in this paper: it shows that any allocation which is incentive-feasible in an environment with money is also incentive-feasible when agents have access to memory. The logic behind this proposition is that with a historical record, we can construct strategies for agents using the direct mechanism in any environment with memory that correspond to what happens in an environment with money. In the monetary environment, when an agent gives up resources today, he receives money which can be used to purchase resources next period. Analogously, in an environment with memory, an imaginary balance sheet is kept for each agent. When an individual gives consumption to someone else, his balance rises, and his capacity for receiving future transfers goes up. When he gets consumption from someone else, his balance falls, and his capacity for receiving future transfers declines. In the monetary environment, money is merely a physical way of maintaining this balance sheet.

The following proposition serves to formalize this logic.

Proposition 3: Any incentive-feasible allocation in an environment with money is an incentive-feasible allocation (of the K perishable goods) in the same environment with memory.

Proof: In Appendix. Δ

The basic intuition of the proof is as described above: in an environment with memory, individuals can react to trading histories that lead to an agent's having a particular amount of money just as they react to his having that money in the environment with money.

2. Applications to Specific Environments

One way to see how Proposition 3 works is through the overlapping generations setup discussed in Section II.2. Suppose each of the initial old agents have 1 token of money. I claim that an allocation is incentive-feasible in this environment with money if and only if it gives (c^y_{jt}, $c^o_{j,t+1}$) to each agent j where

$$
(1) \qquad \begin{aligned}
& u(c^y_{jt}, c^o_{j,t+1}) \geq u(y_1, y_2) \\
& c^y_{jt} \geq 0,\ c^o_{jt} \geq y_2 \\
& c^y_{jt} + c^o_{jt} \leq y_1 + y_2.
\end{aligned}
$$

To see this, suppose that agents use the following collection of strategies in the direct mechanism:

If old agent j has 1 unit of money, young agent j and old agent j write down the above allocation.

If old agent j has ε units of money ($\varepsilon < 1$), young agent j and old agent j write down autarky.

Note that given the conditions on the allocation, these strategies specify best responses. This shows that any allocation in (1) is incentive-feasible. But since any agent can always choose autarky, there are no other incentive-feasible allocations.

Now suppose that memory is added to the overlapping generations environment instead of money. The set of incentive-feasible allocations is exactly the same as in the environment with money. Consider any allocation in (1); the following strategies implement it. If all previous agents labelled j have gone along with the allocation, then the current pair also go along with it. If any agent j has failed to go along with the allocation, then the current pair fails to along with it. Again, because of the restrictions built into (1), these strategies specify best responses in every match, so any allocation in (1) is incentive-feasible in the environment with memory.

Thus, in the overlapping generations setting, an even stronger result than Proposition 3 is true: money is technologically equivalent to memory. But this is an artifact of the simple matching process in the overlapping generations environment. Consider the Trejos-Wright (1995) random matching environment described in Section II.4. In that setting, if full commitment were possible, the efficient allocation would involve type (i+1) agents giving type i agents y^* units of output whenever they are paired, where y^* satisfies $u'(y^*) = 1$.

Using a standard folk theorem argument, it is clear that if β is sufficiently large, this allocation is incentive-feasible in an environment with memory (see Aiyagari and Wallace (1991)). However, the allocation delivers the same *ex-ante* utility after every match. Such an allocation cannot be incentive-feasible in the same environment with money: agents who produce must be offered an increase in future utility relative to those who consume.[6] Hence, in the Trejos-Wright (1995) model, memory dominates money.

3. Robustness

It is useful to try to understand how robust Proposition 3 is. First, note that its proof does not rely on the restriction (R2) that there are no static gains to trade. The only use of this restriction is to make the role of money and memory more dramatic when there is no commitment (as seen in Proposition 2).

The two other major restrictions on the matching process are (R1) and the independence of s_t from its past. Note that the existence of nonautarkic perfect public equilibria uses these properties heavily. For this reason, a major reworking of the theory is probably necessary to understand the relative roles of memory and money in environments in which these restrictions on the matching process are not satisfied.

Williamson and Wright (1993) and Huggett and Krasa (1996) discuss aspects of monetary exchange in environments in which agents are asymmetrically informed about goods quality.

[6]I thank Neil Wallace for this argument.

I conjecture that Proposition 3 in this paper can be extended to environments in which asymmetries of information are *transitory* - that is, environments in which only perishable goods can be lemons, or in which agents have privately observable endowment and/or preference shocks that are i.i.d. over time. Intuitively, the transience of the information differences means that agents' beliefs about this information are not altered by having memory (seeing the past actions of their current trading partners) as opposed to money. However, with *persistent* asymmetries of information, as when agents can produce *durable* lemons, I believe that it is only possible to prove a version of Proposition 3 by changing the definition of memory.[7]

Throughout the paper, money is modelled as intrinsically useless. Suppose money were instead a perfectly durable good that entered preferences separably from the K perishable goods and provided some small amount of utility when consumed. Then, it is possible to extend Proposition 3 so that any incentive-feasible allocation of perishable goods in an environment with money is also incentive-feasible in the same environment with memory.

Proposition 3 defines memory as being the past actions of all agents in $Q_t(\omega)$. It is tempting to think that this much memory isn't necessary: what happens if agents know only the past transfers of resources made and received by all agents in their current match (i.e., $P_t(\omega)$)? It is important to note that in the environment with memory, each individual's imaginary balance sheet does not just depend on his own transfers; in particular, the maker of a transfer of resources does not see an increase in his balance if the receiver's balance was zero. For this reason, the entry on any person's balance sheet is a function not just of his actions, but also those of his trading partners, their trading partners, and so on. Thus, if it is to replicate the

[7] I thank Ned Prescott and Chris Phelan for stressing this point to me.

technological benefits of money, memory must include the past actions of all agents in $Q_t(\omega)$, not just in $P_t(\omega)$.

On the other hand, the proof of Proposition 3 also relies on memory's being *limited*: agent ω only has access to the histories of agents in $Q_t(\omega)$, and not to the histories of all agents. The Appendix contains an example that shows that with the latter, more expansive, version of memory, some incentive-feasible allocations in the environment with money may no longer be incentive-feasible in the environment with memory.[8]

V. Commitment and Memory

The previous sections argue that the technological role of money is merely to keep track of promises of future benefits in exchange for past transfers of resources. Seemingly, though, we could think of all paper assets in the same way. Thus, when I hold a bond, it is a (intrinsically useless) piece of paper that promises me consumption in the future for having given up consumption in the past. Can the role of bonds and stocks be totally subsumed by memory?

The answer is no: asset market equilibria cannot generally be obtained as the PPE

[8]The intuition of the example works as follows. Suppose s_t is deterministic after period t, and let G be the set of agent ω's future trading partners; suppose agent ω knows that all members of G will have the same amount of money. In the environment with money, he doesn't know how much this amount of money will be; suppose he believes that it has a high probability of being zero. Because of his uncertainty, he is willing to give up goods in period (t-1) in exchange for money. If he knows the past of all people, he may know that members of group G will be unwilling to give him resources in the future, and so he may in turn be unwilling to give up real goods today.

Notice that this intuition, and the example in the Appendix, require a matching process that is both deterministic and random. I believe that with matching processes that are either purely deterministic or purely random, and given the independence of s_t over time, Proposition 3 is true even with the more expansive notion of memory.

outcomes of a no-commitment trading mechanism. This is most easily seen in finite horizon settings. In these environments, there is always a unique PPE outcome to any no-commitment trading mechanism: autarky (assuming that there are no static gains to trade). Yet, there are typically nonautarkic bond market equilibria.

Bonds are able to achieve these additional allocations by the use of external punishments that can force agents to violate *ex-post* individual rationality constraints. For example, in a two period setting, the borrower has every incentive in the second period to default on a loan. There must be some threat of external force that makes the borrower abide by the terms of the contract. This threat of external force is not present in monetary transactions. Money only provides memory, but bonds provide both memory and commitment.[9]

The distinction between memory and commitment is also important for understanding the secular trends in the velocity of M1 and the velocity of currency observed in the United States since the end of World War II (for graphic evidence, see page 107 of Barro (1993)). Generally, these secular trends are interpreted as reflecting a greater usage of fully enforceable debt arrangements as opposed to currency in transactions; informally, the economy is said to be becoming more "Arrow-Debreu" over time. In recent papers, Ireland (1991, 1994) and Chatterjee and Corbae (1995) formalize this explanation in two different types of models.

It is undoubtedly true that the trend in monetary velocity indicates that individuals are substituting away from currency towards credit cards and other means of payments. But it is

[9]In infinite horizon settings, bond market equilibria may be incentive-feasible. Then, the distinction between bonds and money becomes murkier. Indeed, Taub (1994) proves in an infinite horizon economy that bonds and money are equivalent when all agents are assumed to have linear utilities and there is no commitment.

important to realize that credit cards are not the full commitment contracts traded in the Arrow-Debreu world. Banks have a limited range of options available in terms of punishing individuals who do not repay their credit card debts; after all, the days of debtor prisons are certainly behind us.

Like money itself, credit cards are primarily mnemonic devices. When I purchase an object on my credit card, I give no real resources in exchange. As with money, I am essentially receiving a "gift". Unlike with money, credit cards allow the information about the transfer of real resources to be cheaply written into a credit history. This credit history can then be easily accessed by others who are my trading partners. According to this argument, the transactions/money ratio is growing over time not because agents have easier access to fully enforceable debt arrangements, but rather because the costs of storing and accessing information are declining rapidly.[10]

VI. Conclusions

This paper demonstrates that in a large class of environments, any technological advantages offered by money are also offered by an alternative technology: memory. Moreover, in at least some environments, memory may technologically dominate money. There are two major implications of these results. First, the paper provides a definitive and general response to the old question of "Why does money exist?" Often, economists answer this question by

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[10]Aiyagari and Williamson (1996) study optimal allocations in an economy in which the planner can keep track of the past through either a "noisy" record of past announcements and transfers by the agents or through their moneyholdings. As in my discussion, they interpret the former as being akin to credit cards. As in Townsend (1989), trade of consumption for "money" in Aiyagari and Williamson (1996) is *in*voluntary.

saying that money is a store of value, money is a medium of exchange and/or money is a unit of account. The message of this paper is that the only thing that money adds to a society is a (limited) ability to keep track of the past.

The second implication is related, but is more "real world": the paper demonstrates that money may only be an imperfect substitute for high quality information storage and access. This "real world" message serves to underscore that the government's monopoly on seignorage might be in some jeopardy as information access and storage costs decline.

In recent years, the pure theory of money (especially following Townsend (1980)) has been concerned with what kinds of "spatial" environments will give rise to the use of currency as opposed to credit. The reasoning in this paper emphasizes that the crucial attribute of a particular spatial arrangement is not the geography itself, but rather the technological limitations to memory and commitment that the geography suggests. Thus, in the turnpike model, it seems "natural" for even and odd agents who meet at a given trading post to be unaware of each others' pasts; this lack of memory generates a need for money. Nonetheless, the lack of memory is not intrinsic to the geographical specification, but rather reflects a particular technological deficiency.

Similarly, in the turnpike model it seems "unnatural" for an odd agent j to give up consumption today for a piece of paper that reads "Agent j in the even cohort at the next trading post to the left owes the bearer 1 unit of consumption." It is the absence of such contracts that makes money valued. Again, though, there is nothing intrinsic to the geography that rules out such contracts; rather, the lack of such contracts reflects the absence of a particular type of enforcement technology.

I believe that there are many possible extensions to the type of analysis in this paper. Here is one route to take. This paper is about two polar extremes: environments with high quality information about the past, and environments with money. In the real world, money and memory co-exist. Kocherlakota and Wallace (1996) examine a particular setting in which society has access to both money and memory that is updated only infrequently. We show that it may be optimal for such a society to use both memory and money in allocating resources among agents.

Appendix

Proof of Proposition 1: In what follows, I consider any PPE allocation of some no-commitment trading mechanism.

(i.) No Money or Memory

For any trading mechanism, the PPE equilibrium specifies a feasible allocation of resources $\mathbf{c}(t, P_t(\omega))$ in each match as a function of time and of the indices of agents involved in the match. In the direct mechanism, I claim that the following is a PPE:

In any match, all agents in the match choose the same feasible allocation $\mathbf{c}(t, P_t(\omega))$, where $\mathbf{c}(t, P_t(\omega))$ is the equilibrium allocation in that match; agent ω chooses some element of $\{y_t^\omega\} + Y_t^\omega$ that delivers momentary utility equal to $u_{aut,t}^\omega$.

Suppose some agent chooses a different action in some match. Then, he gets momentary utility less than or equal to $u_{aut,t}^\omega$. His future utility is unaffected, because his change will not be reflected in public information in future matches. But $\mathbf{c}(t, P_t(\omega))$ delivers at least as much momentary utility as $u_{aut,t}^\omega$, because it is a PPE of a *no-commitment* trading mechanism.

(ii.) Money

Using any trading mechanism, an agent's continuation utility (after any match) in a PPE depends only on time, his index, and on his moneyholdings. A PPE in any mechanism specifies an equilibrium allocation $\mathbf{c}(t, s_t, \mathbf{m}_{t-1})$ as a function of time, agent indices, and moneyholdings.

In the direct mechanism, I claim that the following is a PPE.

In any match, all agents in the match choose the same feasible allocation $\mathbf{c}(t, P_t(\omega), \mathbf{m}_{t-1})$; agent ω chooses some element of $\{y_t^\omega\} + Y_t^\omega$ that delivers momentary utility equal to $u_{aut,t}^\omega$.

Suppose some agent ω chooses a different action in some match. Then, he gets momentary utility less than or equal to $u_{aut,t}^\omega$, and the continuation utility associated with having m_{t-1}^z units of money. The latter is the same as in the original PPE, because, along the equilibrium path, agents are getting the same allocation of resources. But agent ω could have achieved this combination of momentary and future utility in any no-commitment trading mechanism by choosing the right sequence of actions.

(iii.) Memory

Consider a PPE in some trading mechanism. Agent strategies in the direct mechanism work as follows. Suppose allocations in every previous match involving agents in $Q_t(\omega)$ are the same as they would be if agents used the equilibrium strategies from this PPE. Then, agents choose the equilibrium allocation that the original PPE specifies for the match. If not, they choose autarky. This collection of strategies guarantees that if any individual deviates, he gets autarky currently and forever; this is clearly no better than his utility level in the original PPE. Δ

Proof of Proposition 3: Consider an incentive-feasible allocation in an environment with money.

It is the outcome of some PPE when agents use the direct mechanism. In that equilibrium, an agent's action in any match is a function of the identities of the other agents in the match, and the moneyholdings of all the agents in the match.

Now consider the same environment with memory. For now, suppose that at each date t, every agent ω is characterized by a summary statistic $Z_t(\omega)$. Later, I will argue that this summary statistic is redundant, given the information available in the historical record. Define $Z_0(\omega) \equiv m_0(\omega)$ for all ω in Ω_1, so that every agent alive at the beginning of period one has a summary statistic equal to his moneyholdings in the environment with memory.

Define a match's history to be the past actions of all agents in $Q_t(\omega)$. The following describes a response function for agents.

In a match, each individual chooses the same split of the K perishable goods as he would in the money PPE, given the identities of the agents in his match, and treating the summary statistics of the agents in the match as equivalent to their moneyholdings. After the match, individual summary statistics are updated to be equal to the moneyholdings that would occur after the match in the allocation in the environment with money.

Given this definition of response functions, I claim that the following is an equilibrium collection of strategies. Recall that in the direct mechanism, an action is a choice of a feasible allocation and an autarkic consumption vector. Agents use the above response functions in determining the choice of the feasible allocation if the match's history is consistent with all agents in $Q_t(\omega)$ having always used the above response functions in the past. If the match's history is inconsistent with all agents in $Q_t(\omega)$ having always used the above response functions in the past, then all agents choose an allocation that delivers current utility $u^{\omega}_{aut,t}$ to each agent ω in the match.

It is clear that the allocation in the environment with money is the outcome of agents' playing these strategies in the environment with memory. The question remains whether these strategies are actually an equilibrium. Note first that the summary statistics are complicated functions of the history of past actions of all agents in $Q_t(\omega)$ and the initial moneyholdings of the agents in Ω_0. It follows that agents don't actually have to see the summary statistics; the strategies are informationally feasible, assuming that in each match agents have memory.

Next, I need to show that at any information set, all agents receive at least as much utility from the PPE allocation in the environment with money as from any autarkic allocation. But this is trivial, because agents could always choose moneyholdings equal to zero and an autarkic consumption vector. Δ

Example: The following is an example of an environment in which memory of *all* agents' past actions renders incentive-*in*feasible an allocation that is incentive-feasible in the same environment with money.

The environment is an overlapping generations setup in which agents all live four periods. In each cohort, there is a continuum of agents, indexed by [0,1]. (Four cohorts are alive in period one.) Except for one cohort, all agents have the same utility function

$$-y_1 - y_2 - y_3 + u(c_4)$$

where y_i is the amount of output produced in period i of life and c_4 is the amount of consumption in the last period of life. (Agents can only produce output in the first three periods of life.)

One cohort, born at date t, is exceptional. Agents in that cohort have a utility function of the form

$$-y_1 + u(c_2)$$

so they like consumption in their second period of life; they can only produce output in the first period of life.

The matching process works as follows. In each period, the youngest cohort is matched randomly with the old cohort. The two middle cohorts are matched *deterministically*: the agents labelled ω are matched with one another. Note that this process satisfies (R1) and (R2).

The set of individually feasible moneyholdings is $X \equiv \{0,1\}$. Suppose the agents in each cohort are indexed by the set [0,1]. Then, the agents indexed by $\omega \in [0, 1/2]$ in the three initial oldest cohorts are each endowed with one unit of money. The initial youngest cohort is endowed with zero units of money.

Suppose y^* and y^{**} satisfy the inequalities

$$-y^* + u(y^*) \geq 0$$

$$-y^* + u(y^{**})/2 \geq 0$$

$$-y^* + u(y^*) \geq 0.5\{-y^{**} + u(y^*)\}$$

$$-y^{**} + u(y^*) \geq 0.$$

(For example, if $u(x) = x^{1/2}$, then $y^* = 0.15$ and $y^{**} = 0.05$ satisfy these inequalities.) Then, the following is an incentive-feasible allocation in this environment with money. In every period, if an agent in the oldest cohort has a unit of money, the young agent in his match gives him y^* units of output for that unit of money. In period (t+1), in any match in which the agent born in period t has money and the agent born in period (t-1) does not, the agent born in period (t-1) will produce y^{**} units of output in exchange for the unit of money.

The first inequality guarantees that all agents, except those born in period t or in period (t-1), follow the equilibrium strategies when young. The second inequality guarantees that an agent born in period t follows the equilibrium strategy when young; his future utility is divided by two because he has a probability of 1/2 of meeting someone next period who will not accept his money. The third inequality guarantees that in period (t-1), agents born in period (t−1) will buy the money from those old agents who have it rather than wait to buy money from the agent born in period t - if he has it! The final inequality guarantees that in period (t+1), agents born in period (t-1) without money will accept it from agents born in period t who have it.

Now suppose the typical agent ω has memory of the past events in the lives of all agents, not just those agents in $Q_t(\omega)$. Then, the above allocation is no longer incentive-feasible. Given this kind of memory, in period t, the youngest cohort sees the results of all matches in period (t-1). This means that instead of all of the youngest cohort thinking that there is a probability 1/2 of their trading partner next period not producing, half of the youngest cohort knows that they will be matched with someone who will not produce next period. As a consequence, half of the youngest cohort refuses to produce y^* units of output for the oldest cohort.

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